Unit Interval Editing is Fixed-Parameter Tractable

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Graph modification problems

For every graph class $\mathcal{G}$, we can study:

Definition (Graph modification problem)

Input: a graph G of size n and a nonnegative integer k

Task: find $\leq k$ modifications that transform G into a graph in $\mathcal{G}$?

Typical modification operations:

- deleting edges,
- adding edges, or
- deleting vertices.

Combined modification operations:

- deletion: vertex deletions and edge deletions;
- edge editing: edge additions and edge deletions;
- editing: all three modifications.

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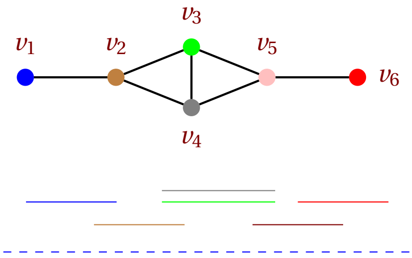
In other words, the question is if G belongs to the class

- $\mathcal{G} + ke$: a graph from $\mathcal{G}$ with k extra edges;
- $\mathcal{G} - ke$: a graph from $\mathcal{G}$ with k missing edges;
- $\mathcal{G} + kv$: a graph from $\mathcal{G}$ with k extra vertices.

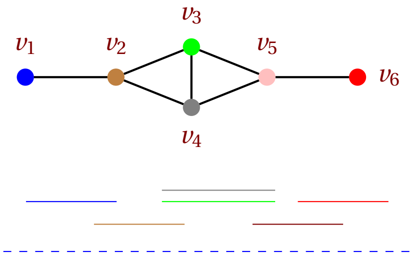
Unit interval graphs

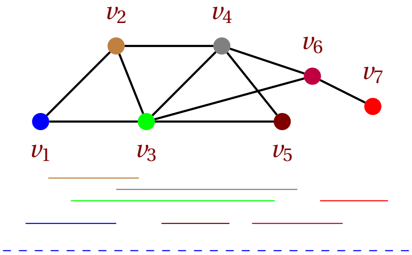


Definition

There are a set of unit-length intervals $\mathscr{I}$ on the real line and $\phi : V \to \mathscr{I}$ such that $uv \in E(G)$ iff $\phi(u)$ intersects $\phi(v)$.

Unit interval graphs





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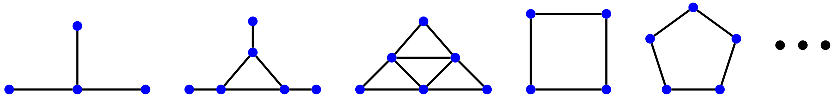

(e) claw (f) net (g) sun (h) C_4 (i) C_5

unit interval ⊂ interval ⊂ chordal (hole-free)

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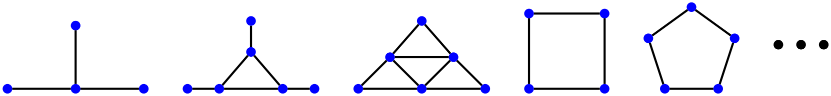

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Previous work

- [Kaplan et al. (FOCS'94, SICOMP'99)] showed unit interval completion is FPT.
- [Cai '96] gave a better analysis, $O(4^k \cdot (n+m))$.
- [Marx (WG'06, Algorithmica'10)] showed Chordal Deletion Is FPT, implying.
- [van Bevern et al. '10] gave a direct algorithm (iterative compression).
- [Villanger (IPEC'10, Algorithmica'13 with van't Hof)] showed
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Standard technique

A small subgraph F can be found in $n^{|F|}$ time and dealt with an $|F|$-way branching.

- **completion**: the approach of chordal completion works here
  The number of ways a ℓ-hole can be triangulated is exactly the $(\ell - 2)$nd Catalan number $C_{\ell-2}$, which is at most $4^{\ell-3}$. [Kaplan et al. '94; Cai 96].
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Unit interval vertex deletion

Theorem (van Bevern et al. '10)

The disjoint version of unit interval vertex deletion can be solved in $O^*((14k)^k)$ time on $\{claw, net, tent, C_4, C_5, C_6\}$-free graphs.

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Unit interval vertex deletion can be solved in linear time on $\{claw, net, tent, C_4, C_5, C_6\}$-free graphs.

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Unit interval vertex deletion remains NP-hard on $\{claw, net, tent\}$-free graphs.

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the brute-force used in dealing with small subgraphs induces a large polynomial factor (n^6) in the running time.

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(Proper) circular-arc graphs

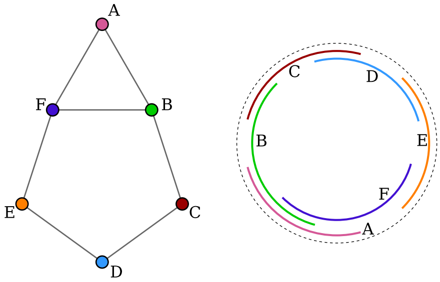
Theorem (Villanger '13)

A $\{claw, net, tent, C_4, C_5, C_6\}$-free graph is a proper circular-arc graph.

Definition (Circular-arc graphs)

A graph is a circular-arc graph if there are a set $\mathscr{A}$ of arcs on a circle and $\phi : V \to \mathscr{A}$ such that $uv \in E(G)$ iff $\phi(u)$ intersects $\phi(v)$.

proper: no arc properly contains the other.



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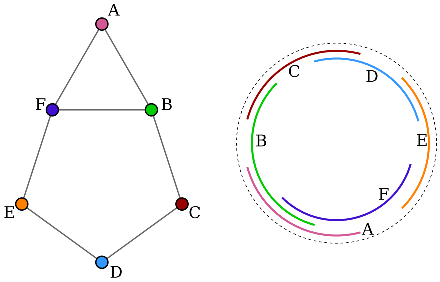

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Proper Helly circular-arc graphs

Helly: Any set of pairwise intersecting arcs has a common point.

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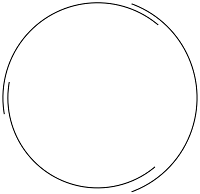
A graph having an arc model that is both proper and Helly.



A bad example

A Helly model

net

A proper model

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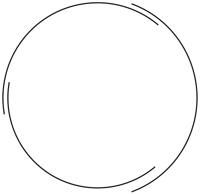

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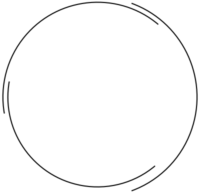

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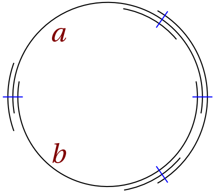

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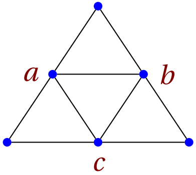

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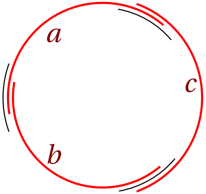
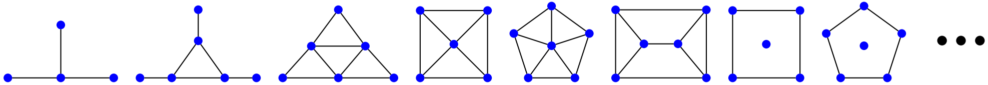


A proper model

Why proper Helly?

Theorem (Tucker '74; Lin et al. 13)

A graph is a proper Helly circular-arc graph if and only if it contains no claw, net, tent, W_4, W_5, $\overline{C_6}$, *or* C_ℓ^* *for* $\ell \geq 4$ *(a hole* C_ℓ *and another isolated vertex).*



A trivial corollary

If a proper Helly circular-arc graph is chordal, then it is a unit interval graph.

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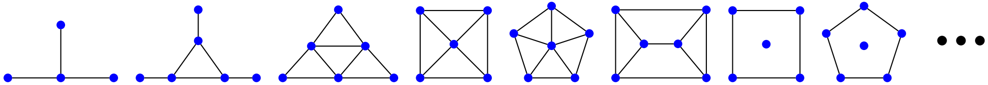

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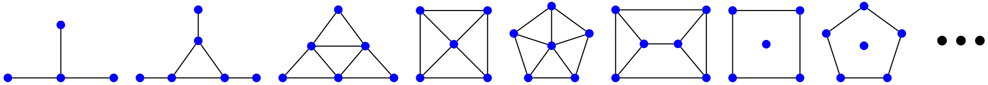

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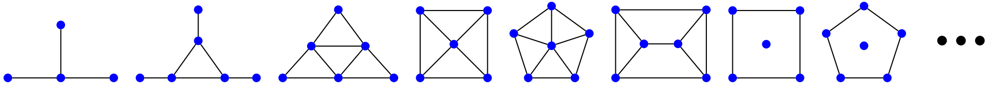

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{Claw, net, tent, C_4}-free graphs

Main structual theorem

Let G be a connected graph.

1. If G is $\{\text{claw}, \text{net}, \text{tent}, C_4\}$-free, then it is either a fat W_5 or a proper Helly circular-arc graph.
2. In $O(m)$ time we can
   - detect an induced claw, net, tent, C_4 of G,
   - partition $V(G)$ into six cliques constituting a fat W_5, or
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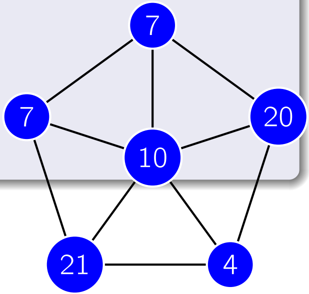

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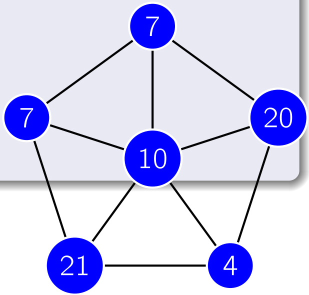
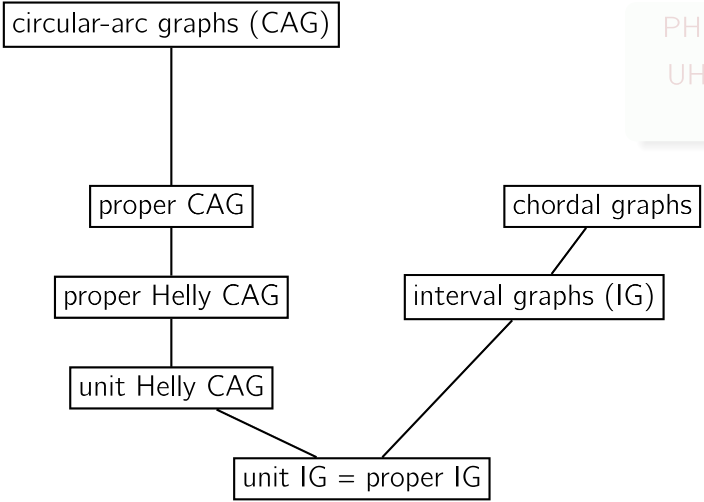


Big picture



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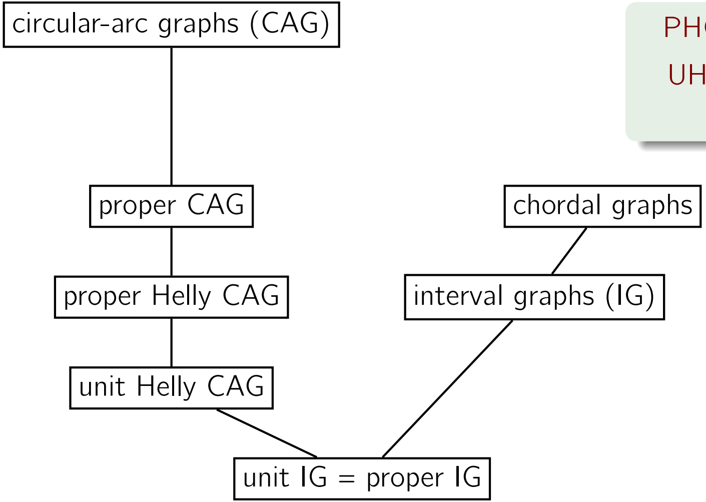

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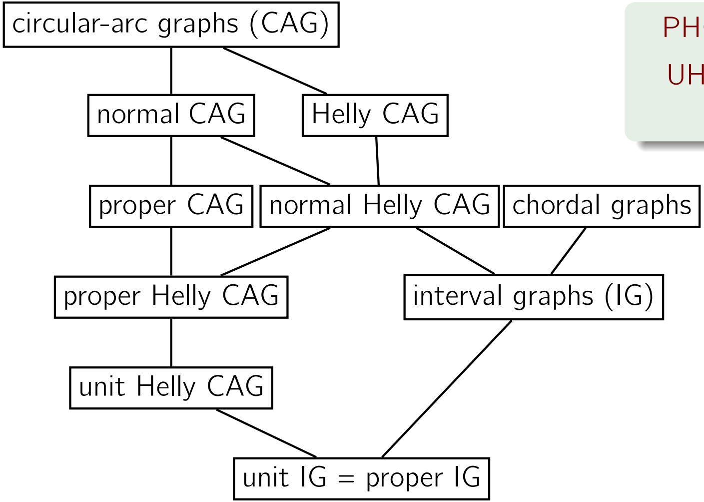

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How about unit Helly circular-arc graphs?

This is actually the CI(ℓ,1) graph defined by [Tucker '74]; see also [Lin et al. '13].

Remark

Therefore, proper Helly circular-arc graphs are the best we can expect in this sense.

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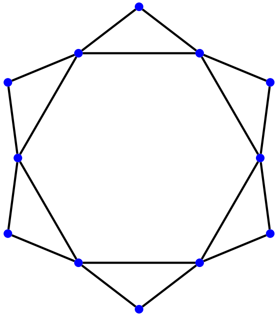

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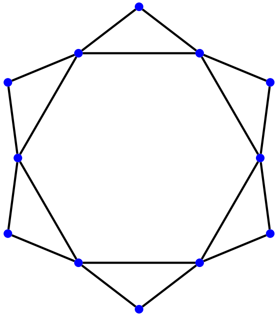

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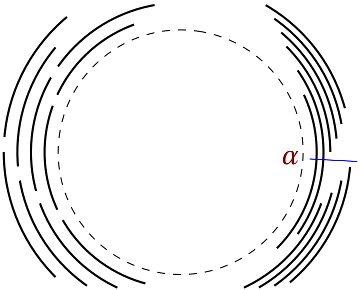

easy for proper Helly circular-arc graphs.

trivial for fat W_5's.

Results

an $O(6^k \cdot m)$-time parameterized algorithm; and

an $O(nm)$-time approximation algorithm of ratio 6.

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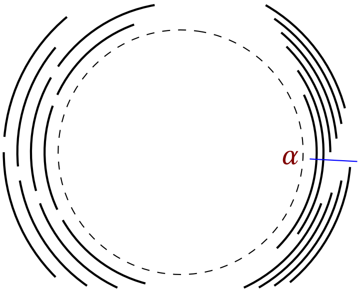

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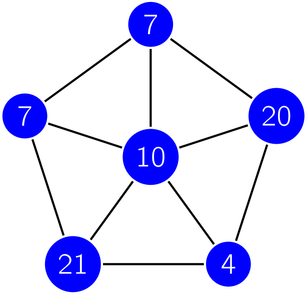

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Results

an $O(6^k \cdot m)$-time parameterized algorithm; and

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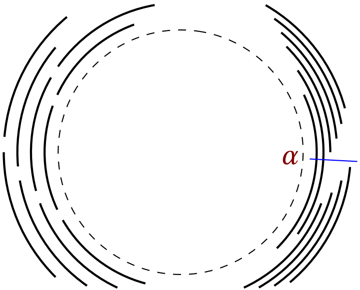
Vertex deletion: the appetizer



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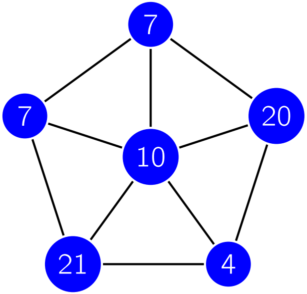

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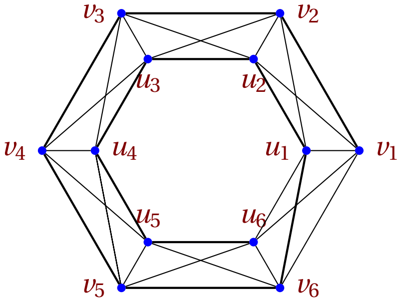

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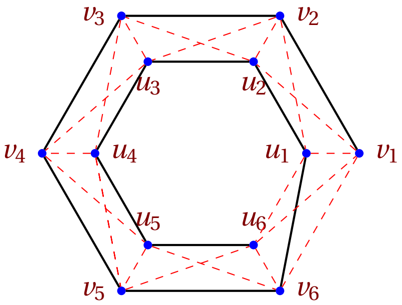
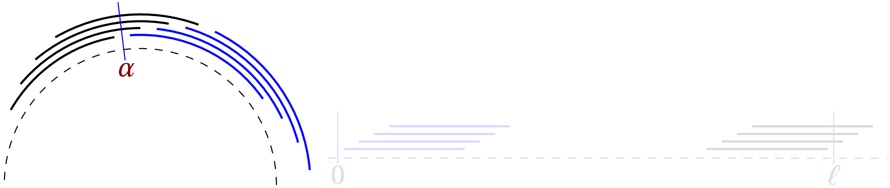


To break long holes

Definition

$$\overrightarrow{E}(\alpha) = \{vu : \alpha \in A_v, \alpha \notin A_u, v \to u\},$$

where $v \to u$ means that arc A_v intersects arc A_u from the left.



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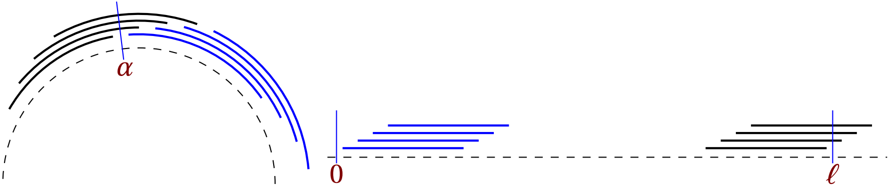

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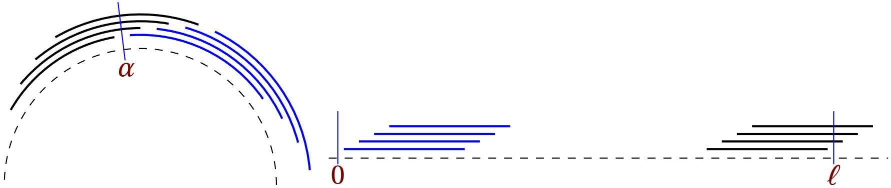

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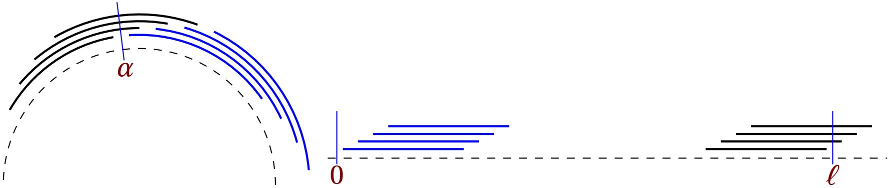

A nontrivial corollary

Any minimum solution is $\overrightarrow{E}(\alpha)$ for some point α.

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- Both deletion problems reduce to find a weakest point.
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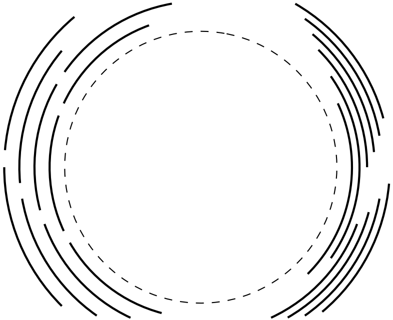

- it suffices to try $2n$ different points (n actually).
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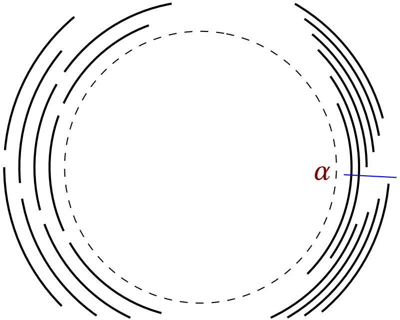

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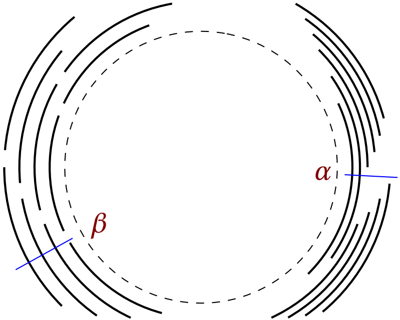

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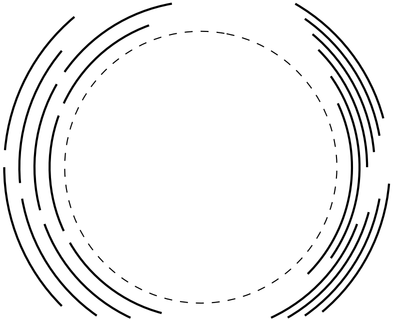

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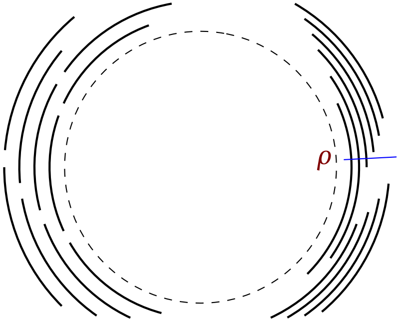

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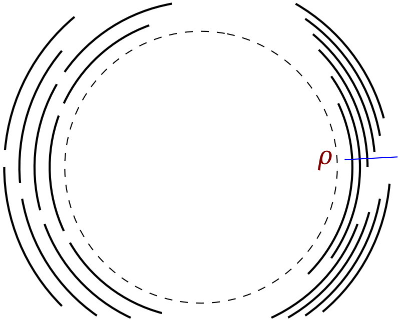

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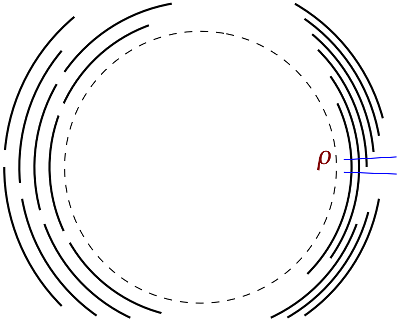

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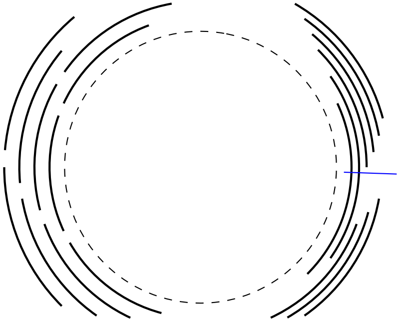

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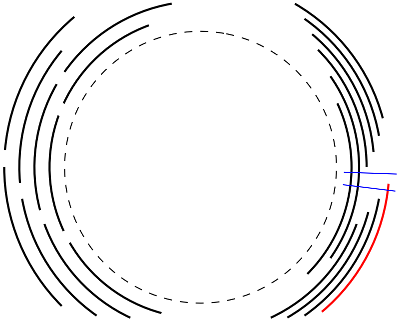

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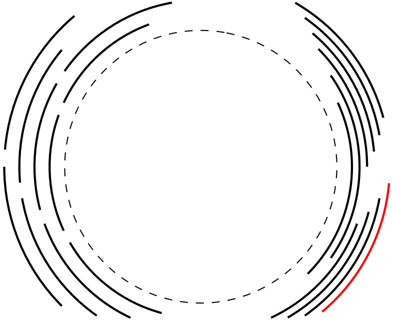

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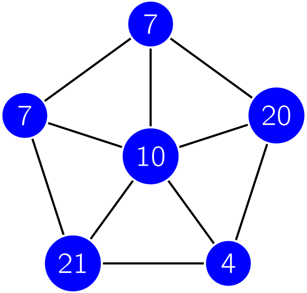

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An $O(9^k \cdot m)$-time algorithm for unit interval edge deletion $\Rightarrow O(4^k \cdot m) \Rightarrow O(3.xx^k \cdot m)$.

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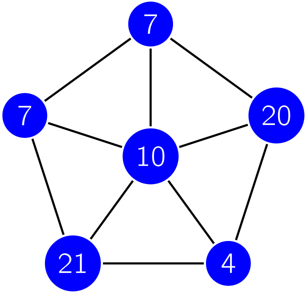

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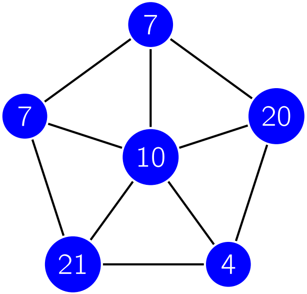

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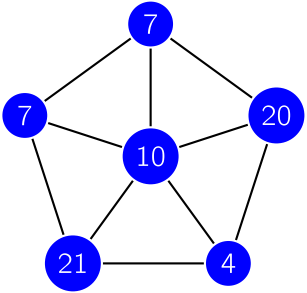

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The editing problem

Definition

(V_-, E_-, E_+) is an editing set of G if the deletion of E_- from and the addition of E_+ to $G - V_-$ create a unit interval graph.

The unit interval editing problem: Is there an editing set such that $|V_-| \leq k_1$ and $|E_-| \leq k_2$ and $|E_+| \leq k_3$.

We use $k := k_1 + k_2 + k_3$ as the parameter.

Remark

This is different to ask for "at most k modifications" to make G a unit interval graph:

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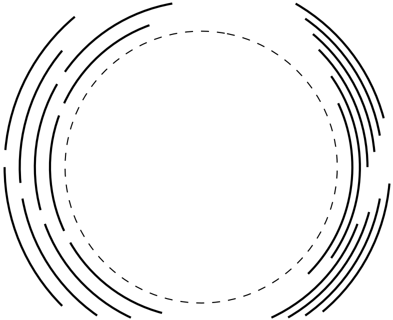
Phase I: reduction

A graph is called reduced if it contains no claw, net, tent, C_4, C_5, or C_ℓ with $\ell \leq k_3 + 3$.

1. find a claw, net, tent, C_4, C_5 if there is one.
2. find a shortest hole H from the remaining proper Helly circular-arc graph.
3. if its length is less than $k_3 + 4$, then branch on $O(|H|)$ ways of dealing with it, and recurse.

After that all forbidden subgraphs are long holes (of length at least $k_3 + 4$).

Phase II



Now all the holes are longer than $k_3 + 4$, only breakable by deletions.

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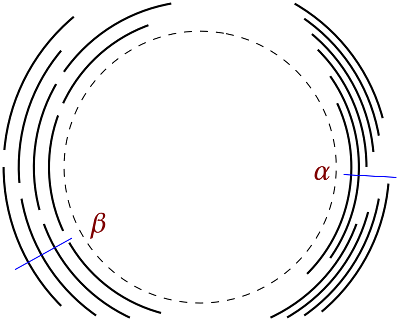

If there is Achilles' heel, we solve it. e.g., if $k_1 \geq 2$, we take α; if $k_2 \geq 6$, we take β;

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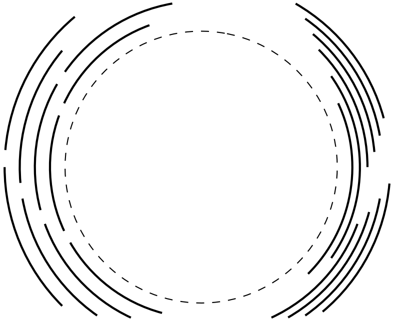

Otherwise, we need to delete both vertices and edges.

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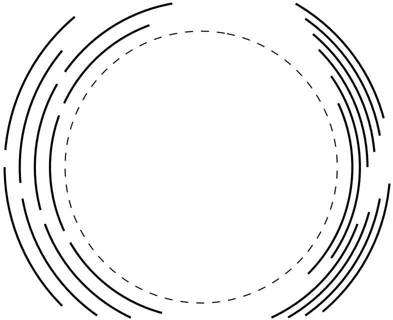

Main observation: after the deletion of V_{-}, it reduces to the edge deletion problem .

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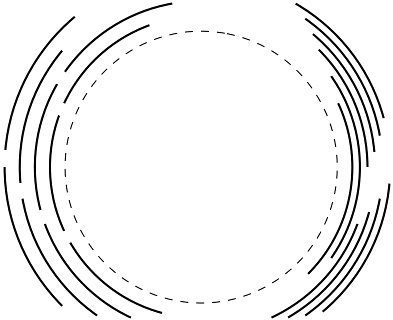

We are looking for a weakest point in $G - V_{-}$, but we don't know where V_{-} is.

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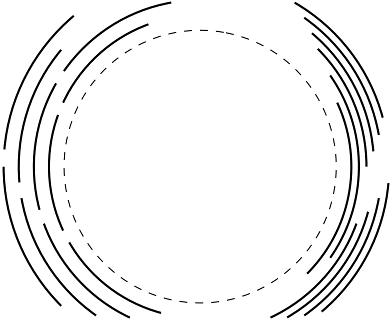

E_- must be local to some point, and V_- has to be local to the same point as well!

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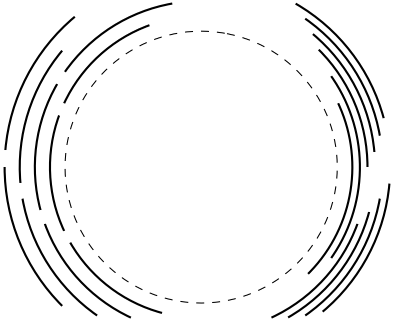

V_- should be chosen to be those incident to the most number of crossing edges.

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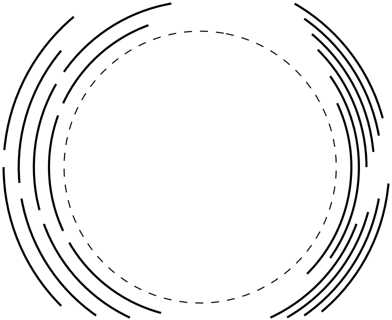
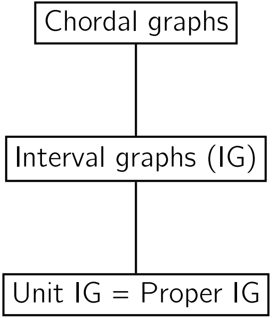


With the similar idea, by scanning the model once, we can find a combined weakest point in linear time. ⇒ an $O((k_3+1)^k \cdot m)$-time algorithm.

Open problems



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- a polynomial kernel for interval vertex deletion?
- a reasonably small kernel for unit interval vertex deletion?
- a subexponential-time algorithm for unit interval edge deletion?
- ETH lower bound of chordal/(unit) interval completion problems.

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[Marx, O'Sullivan, & Razgon, '13; C. & Marx '15; this work.]

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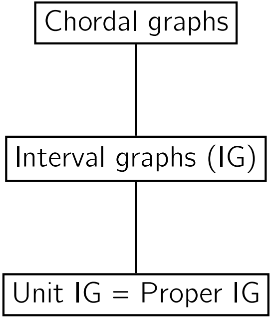

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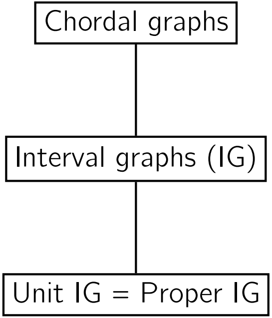

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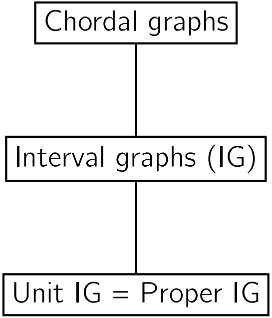

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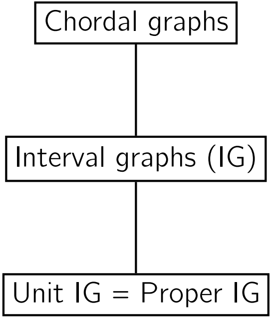

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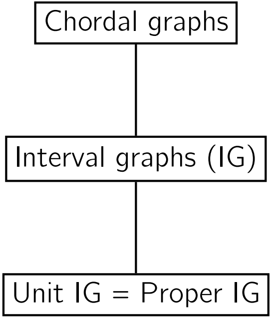

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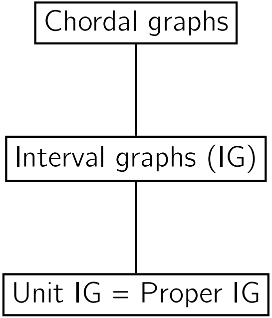
[Marx, O'Sullivan, & Razgon, '13; C. & Marx '15; this work.]

Open problems



- a polynomial kernel for chordal vertex deletion?
- a polynomial kernel for interval vertex deletion?
- a reasonably small kernel for unit interval vertex deletion?
- a subexponential-time algorithm for unit interval edge deletion?
- ETH lower bound of chordal/(unit) interval completion problems.

study of mixed separators: to separate s, t by deleting both vertices and edges;
[Marx, O'Sullivan, & Razgon, '13; C. & Marx '15; this work.]

Thank you!